Finite Range Effects in Knockout Reaction

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Abstract

Finite Range Distorted Wave Impulse Approximation calculations for $(\alpha, 2\alpha)$ reactions have been performed for the first time to remove huge inconsistencies obtained earlier in conventional Zero Range analyses of these reactions. Upto 160 MeV the α–α interaction is observed to have a short range repulsion. Beyond 160 MeV the interaction changes to an all through attractive potential. The knockout reaction's sensitivity to the short distance behaviour of the α–α interaction, indicates it's utility as a probe of the knockout vertex at short distances. It paves the way to analyse the *(e,2e)* and *(n, 2n)* and *(p, pn)* reactions for the (ADS), heavy cluster knockout reactions and Core knockout reactions in the RIB investigations

1. Introduction

The analysis of α-cluster knockout reactions by proton and α- projectiles using the conventional zero range distorted wave impulse approximation (ZR-DWIA) has resulted in large mutual inconsistencies[1- 4]. While the absolute cross section predictions for the $(p, p\alpha)$ reactions are close to the experimental data [1, 5] the corresponding comparison for the $(\alpha,2\alpha)$ reactions lead to almost two orders of magnitude lower predictions [1]. Similar inconsistencies were detected in the case of the knockout of *d*, *t*, and ^3He clusters [6]. Exceptions to these observations, however were seen for the $(\alpha,2\alpha)$ reactions on ^9Be [7] and ^{12}C [8] around 200 MeV. The predictions of small absolute cross sections and hence large α-cluster spectroscopic factors from the analysis of the $(\alpha, 2\alpha)$ reaction data up to ~140 MeV were ascribed to the induced α-clustering, simulated by using large bound state potential radius[2], or in terms of reduced optical distortion effects[9]. However, these ad hoc prescriptions can not account for the data around 200 MeV [7, 8].

In the conventional DWIA treatment of the knockout transition matrix element, the factorization of the knockout vertex contribution is built in. This factorization can arise either from the use of the zero range nature of the knockout vertex transition operator or from the optical distortion free scattering states. While the ZR-DWIA calculations exhibit large optical distortion characteristics [3, 4], the cluster knockout data indicate little influence from these optical distortions.

2. Entrance Channel Cluster Folding Potentials

In the conventional ZR-DWIA analyses, there exist some uncertainties about the entrance channel distorting potentials. The entrance channel optical potential is strictly, the potential for the scattering of the incident particle from the residual nucleus which is to be averaged over the volume of the target nucleus [1]. Such a potential is not obtainable from any realistic experiment. Hence most of the conventional DWIA calculations used potentials which reproduced the scattering data on the target nucleus but with a scaling down factor [1] equal to the ratio of the mass numbers of the residual nucleus and the target nucleus, *i .e* $B/A * V_{aA}$ [Fig.1]. This procedure has been adopted in almost all the cluster knockout DWIA calculations and has been found to be reasonably satisfactory because no significant shape changes in the DWIA l=0 knockout results were witnessed even when the entrance channel potentials are changed substantially.

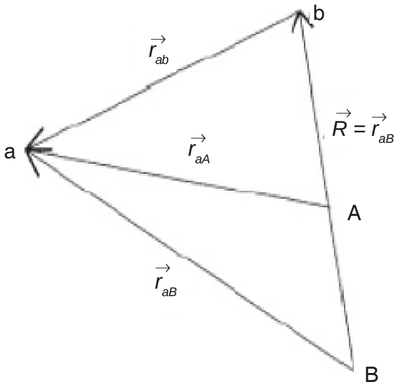

Fig.1 Schematic vector diagram for the cluster folding model.

The cluster folding model potentials have been evaluated for the entrance channel of the $(\alpha, 2\alpha)$ reactions on ^7Li[10]. For the $A(a, ab)B$ knockout reaction, in general, the effective interaction of the incident particle, a with the target nucleus, A is evaluated in terms of its interactions with the residual nucleus, B and the struck particle b, in this single folding model. These interactions are folded over the density distribution of the target, which we have approximated by the square of the ground state inter-cluster radial wave function. Thus this interaction is:

$$V_{aA}(\vec{r}_{aA}) = \int[t_{ab}(\vec{r}_{aA} - \frac{B}{A}\vec{R}) + t_{aB}(\vec{r}_{a} \quad .$$

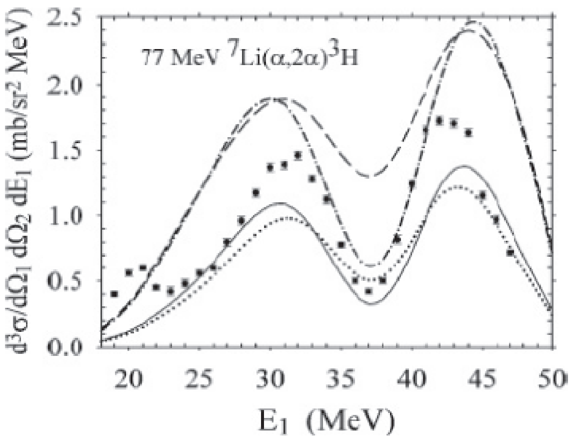

Fig.2 The DWIA calculations using various entrance channel potentials, (- - -) Warner et al, (_._._.) 0.7 times Warner et al, (. . . .) folded V_{aA} and (____) 0.8 times folded V_{aA} compared with the 77 MeV $^7Li(\alpha,2\alpha)^3He$ reaction data.

As the a-b interaction is completely accounted to all orders by the use of the corresponding knockout t-matrix in the DWIA, therefore it has to be dropped from here. Therefore the first term is to be neglected while calculating the entrance channel distorting potential. In order to evaluate the above integral we take $v(\vec{k})$ as the Fourier transform of $V_{aB}(\vec{r}_{aB})$. In terms of which the equation for the entrance channel optical potential reduces to,

$$V_{Ent}(\vec{r}_{aA}) = \iint e^{-i\frac{b}{A}\vec{k}.\vec{R}} \rho(R) d\,\vec{\mathrm{R}}\mathrm{e}^{-i\vec{k}.\vec{r}_{aA}}$$

Using these entrance channel optical potentials from the folding model prescription, the dip at the zero recoil momentum position in the $l \neq 0$ $(\alpha, 2\alpha)$ knockout reactions were seen to be still filled up in comparison to the experimental results [Figs.2,3]. It is also found that when the entrance channel potential depths, both in terms of $B/A * V_{aA}$ as well as from the folding criterion, are reduced by about 25%, the peak to dip ratios increase sharply. This can be found to arise mainly due to large relative changes in the dip cross sections in comparison to only marginal changes in the peak cross section values. Surprising thing is that the peak to dip cross section ratios are almost unchanged while changing from $B/A * V_{aA}$ criterion to the folding criterion. This result is not unique to 77 MeV $^7Li(\alpha,2\alpha)^3H$ reaction alone as the $^7Li(\alpha,2\alpha)^3H$ reaction at 119 MeV and at other energies and for other cluster knockout reactions with $l \neq 0$ bound clusters [4] also show similar behavior. It is to be conceived now that this result is due to a delicate balance of the destructive interference of entrance and exit channel distorted waves which produces the dip in the $l \neq 0$ spectra at the zero recoil momentum position.

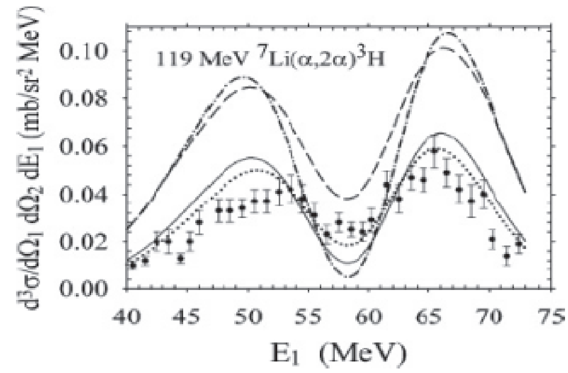

Fig.3 Same as Fig.2 but for 119 MeV ^{7}Li(α, 2α)^{3}He data.

It is thus observed that the discrepancy in the peak to dip cross section ratios in the conventional DWIA predictions of $l \neq 0$ spectra is a result of the uncertainties in the choice of the entrance channel potentials. In fact the entrance channel potentials can be obtained through a consistent procedure of folding the exit channel potentials over the density distribution obtained from the inter-cluster wave function. The use of the folded potentials, however, is seen to reduce the DWIA predictions by a factor of ~ 2 in the $^7Li(\alpha,2\alpha)^3H$ reactions. These results of the entrance channel optical potentials are found to be very different from that obtained using the $B/A*V_{aA}$ criterion. The conventional $B/A*V_{aA}$ potentials of the entrance channel are not appropriate for the DWIA analyses of cluster knockout data particularly for $l \neq 0$ knockout. Besides all this the cluster folding results appear to be more consistent and aesthetically satisfying.

2.2. The FR-DWIA analysis of $(\alpha,2\alpha)$ reactions

The proper finite range (FR)-DWIA formalism for the quasi free knockout reaction has been written down for the first time as described below. The transition amplitude, T_{fi} the knockout reaction $A(\alpha, 2\alpha)\,B$ in the FR-DWIA formalism from the initial state, *i* to the final state, *f* is written as,

$$\frac{d^3\sigma^{L,J}}{d\Omega_1 d\Omega_2 dE_1} = F_{kin}.S_{\alpha}^{LJ}.\sum_{\Lambda}\left|T_{fi}^{\alpha L\Lambda}(\vec{k}\right.$$

Where J and $L(\Lambda)$ are the total and orbital (its azimuthal component) angular momenta of the bound α-particle in the target nucleus, F_{kin} is a kinematic factor and S_{α}^{LJ} is the cluster spectroscopic factor. The exact transition matrix element for the knockout reaction, $T_{fi}^{\alpha L\Lambda}(\vec{k}_f,\vec{k}_i)$ using $t_{12}(\vec{r}_{12})$,the finite range $\alpha-\alpha$ t-matrix effective interaction, is given by

$$T_{fi}^{\alpha L\Lambda}(\vec{k}_f,\vec{k}_i) = \int \chi_1^{(-)*}(\vec{k}_{aB},\vec{r}_{aB})\chi_2^{(-)}$$

$$\times\ t_{12}(\vec{r}_{12})\chi_0^{(+)}(\vec{k}_{1A},\vec{r}_{1A})\varphi_{L\Lambda}(\vec{R}_{2B})d\vec{r}_{12}d$$

The distorted waves χ_0, χ_1 and χ_2 of the above equation are evaluated using the optical potentials for the $\alpha_1 - A$, $\alpha_1 - B$ and $\alpha_2 - B$ respectively. Finally all the relative coordinates are expressed in terms of $\vec{r}_{12}(\equiv \vec{r}\,)$ and $\vec{R}_{2B}(\equiv \vec{R})$. While using the PWIA the transition matrix element, T_{fi} of the above equation in factorized in to integrals over $\vec{r}$ and $\vec{R}$ separately. The same is not possible when one uses the full finite range $t_{12}(\vec{r})$ due to the presence of optical distortions. This is because in the FR-DWIA formalism the chosen relative coordinates $\vec{r}$ and $\vec{R}$ get coupled through the distorted waves $\chi_0^{(+)}(\vec{k}_{1A},\vec{r}_{1A})$ and $\chi_1^{(-)*}(\vec{k}_{1B},\vec{r}_{1B})$. The effective interaction operators $t^{\pm}$ used in the FR-DWIA formalism are defined as [11,12],

$$t^{\pm} = V\,\Omega^{\pm}.$$

The Moller wave operator $\Omega^{\pm}$ is such that it transforms the plane wave states, Φ into scattering states, $\psi^{\pm}$ is defined in terms of radial scattering solutions $u_l(kr)$ as:

$$\psi_{\alpha\alpha}^{\pm}(\vec{r}) = \sum_{l=0,2,4} i^l(2l+1)\frac{u_l(kr)}{kr}e^{i\sigma_l}$$

For symmetric $\alpha-\alpha$ particle system only even partial waves contribute.

$$t_{\alpha\alpha}^{+}(E,\vec{r}) = e^{-ikz}\sum_{l=0,2,4...} V_l(r)i^l(2l+$$

With this the final equation for the t-matrix effective interaction becomes

$$t_L(E,r) = \frac{2L+1}{2}\sum_{l,m}V_l(r)i^l(2L+1$$

$$\times\ (2m+1)e^{i\sigma_l}\int_{-1}^{+1}P_L^*(\cos\theta)P_l(\cos\theta)P_m(\cos\theta)d(c$$

Using these the FR-DWIA analyses have been performed for the $(\alpha, 2\alpha)$ data on ^{9}Be at 197 MeV and 140 MeV, on ^{12}C at 200 MeV and 140 MeV and on ^{16}O at 140 MeV and 90 MeV [13]. For these analyses the $\alpha-\alpha$ t-matrix effective interactions obtained from the two types of $\alpha-\alpha$ optical potentials[11,12], one attractive with repulsive core (R+A) and other, all through attractive (A) have been used (see Fig. 4). These two types of $\alpha-\alpha$ optical potentials with same phase shifts have been employed to explain the $\alpha-\alpha$ elastic scattering data as

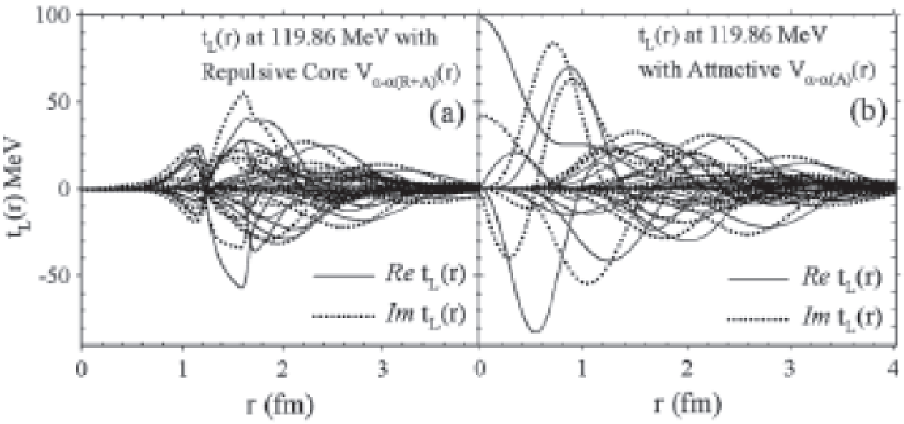

Fig.4 Effective $\alpha-\alpha$ t-matrix interaction $t_L(r)$ vs r at 119.8 MeV for many L values, (a) using $V_{l,a-a}(r)$ with repulsive core and a longer range attraction, (b) using purely attractive $V_{l,a-a}(r)$

well as in the present FR-DWIA analysis of the $(\alpha, 2\alpha)$ reaction. The Finite Range DWIA theory was formulated and the computer code was written in order to evaluate the complex 6-dimensional integral, which factorizes into two 3-dimensional integrals in the Plane Wave Impulse Approximation (PWIA)[13]. The ZR-DWIA theory was under predicting the cross section below ~160 MeV by two orders of magnitude. Whereas above ~160 MeV the ZR-DWIA predicts the $(\alpha, 2\alpha)$ reaction cross section nicely. Here the double differential cross sections were calculated for the first time using the FR-DWIA formalism with repulsive core plus attractive potential below ~160 MeV and only attractive potential above ~160 MeV, and were found to be matching with the structure calculations[13]. The results are shown in Figs.[5-7] along with their spectroscopic factors.

The Finite Range calculations exhibit dramatic change in the cross sections by incorporating the repulsive core potential for energies below ~160 MeV and attractive potential for energies above ~160 MeV. Below ~160 MeV the four neutrons(n) and four protons(p) of the two α-particles can exist in an overlapping position if the two n's and two p's of one α-particle are in the lowest $1s_{1/2}$ shell model state and the other two n's and two p's of the other α in the next shell model state ($1p_{3/2}$, which is situated ~ 21 MeV above the $1s_{1/2}$ shell model state of the α-particle). The total energy of this overlapping system, $E_{\alpha-\alpha}$ will thus be ~4×21=84 MeV (corresponding to the lab energy ~2×84=168

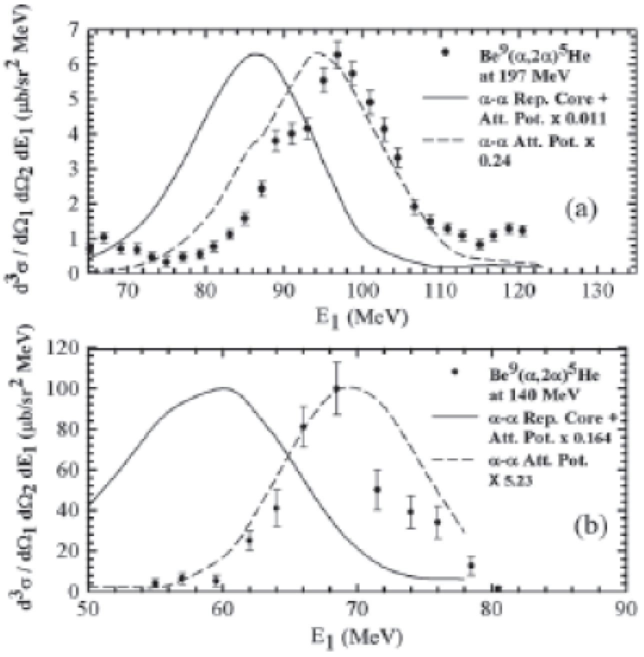

Fig.5 Comparison of $^9Be(\alpha, 2\alpha)$ data with the FR-DWIA calculations using $\alpha-\alpha$ interaction with purely attractive (A) and having a repulsive core (R+A), for (a) 197 MeV, (b)140 MeV.

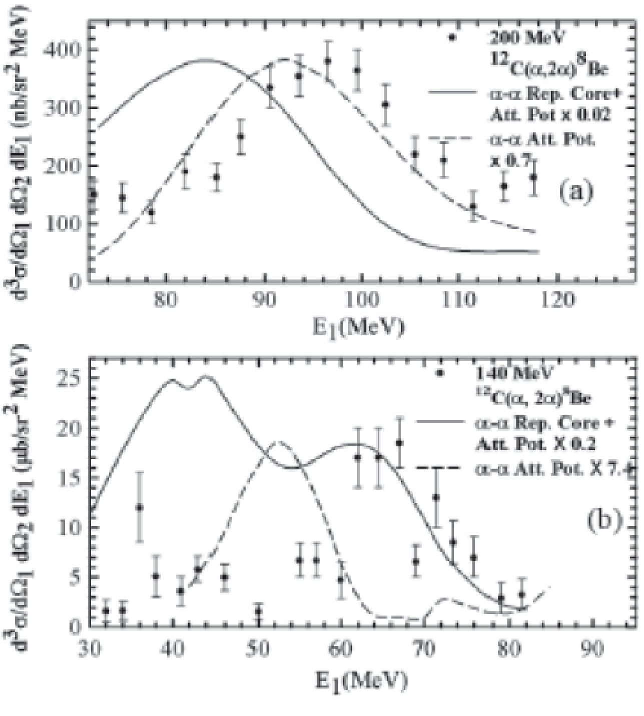

Fig.6 Same as Fig. 5 but for reaction, (a) for 200 MeV and (b) for 140 MeV.

MeV). Thus below this energy, (E_α ~168 MeV) the two α 's would find it energetically more favorable to avoid their overlap with a repulsive core in their interaction. Above this energy, however the two α 's have no such restriction and find it energetically favorable to overlap and can have the usual attractive force between them. This understanding of the change in the nature of the $\alpha-\alpha$ interaction is clearly validated by the present FR-DWIA analyses of the $(\alpha, 2\alpha)$ data. The energy dependence, and the effect of the projectile struck cluster combination on the knockout reaction has been answered nicely in the present non pertrubative study.

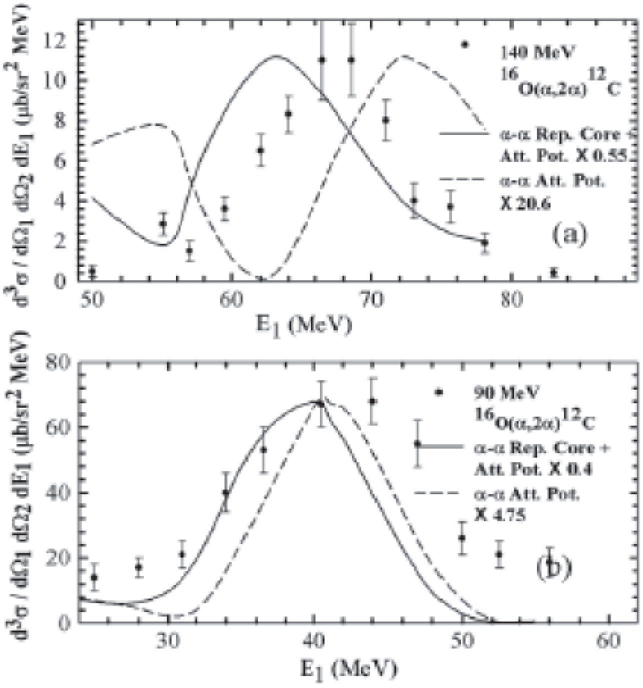

Fig.7 Same as Fig. 5 but for $^{16}O(\alpha, 2\alpha)^{12}C$ reaction, (a) for 140 MeV and (b) for 90 MeV.

2.3. Heavy Cluster Knockout: The $^{16}O(^{12}C,2^{12}C)^4He$ Reaction:

The findings of the $\alpha-\alpha$ t-matrix, $t_{\alpha-\alpha}(\vec{r})$ revealed that it contains a large number of multipole moments as well as it is a very long ranged function of *r* [11]. Large distortions coupled with the finite range of $t_{\alpha-\alpha}(\vec{r})$ lead to the break down of the factorization approximation in the conventional ZR-DWIA [13]. As the ^{12}C nucleus is almost 1.5 times larger than the α-particle, the range of the ^{12}C–^{12}C t-matrix, $t_{C-C}(\vec{r})$ is also expected to be of even more compared to the $\alpha-\alpha$ t-matrix by similar amount. In the two reactions, $^{16}O(a,2a)^{12}C$ and $^{16}O(^{12}C,2^{12}C)^4He$ the inter-cluster wave function as well as the distorting potentials are basically the same but the main difference appears in their knockout vertices. Hence the (C, 2C) reaction, if the finite range nature of the knockout vertex is unimportant, then the extracted spectroscopic factor should be the same for the two reactions [14].

Based on the FR-DWIA theory prediction, the heavy cluster knockout, experiment $^{16}O(^{12}C,2^{12}C)^4He$ was performed at the pelletron-LINAC Facility (PLF), Mumbai. Here the predicted longer range of the C-C *t*-matrix as compared to the $\alpha-\alpha$ *t*-matrix with its longer ranged repulsive core leads to an enhancement of the cross section.

The experiment uses the 118.8 MeV ^{12}C beam on ^{16}O target in the form of 175 $\mu g/cm^2$ of WO_3 supported by 230 $\mu g/cm^2$ of *Au*. The WO_3 was sandwiched by coating another layer of 50 $\mu g/cm^2$ of *Au* on the other side, this is to avoid the pealing off of the WO_3 because of its hygroscopic nature. Two $\Delta E-E$ telescope were mounted at an angle pair of 41^0 and 45^0. A Monitor detector was mounted at an angle of 16^0 with the incident beam. The two ^{12}C's were identified and observed in coincidence and the clean summed energy spectrum of knockout events were recorded as seen in Fig.8. The two $^{12}C_{g,s}$ summed energy peak was observed at 110 MeV corresponding to a Q-value of -7.4 MeV. Corresponding energy sharing

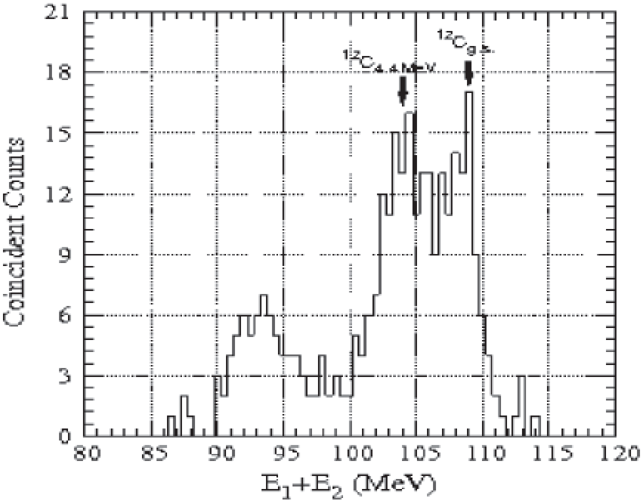

Fig.8 Summed energy spectra for the $^{16}O(\alpha, 2\alpha)^{12}C$ reaction.

distribution was generated from this summed energy spectrum (Fig.9). The $^{16}O(C,2C)^4He$ reaction data was analysed using the FR-DWIA formalism with two drastically different C-C optical potentials which fits the elastic scattering data well. One of these optical potential is totally attractive (A) while the other has a Repulsive core and an Attractive Potential (R+A). The FR-DWIA analysis has been performed using these two. Shapewise one can not differentiate between the two but the absolute cross sections (in $\mu b / Sr^2 MeV$) using potential (A) and potential (R+A) peak values are 38, 445 respectively as compared to the experimental value of 125 ± 50. Clearly the attractive C-C potentials (A) leads to a vary large spectroscopic factor, S_α~3.3 while the (R+A) gives a S_α value ~0.28, much closer to the structure theoretical estimate of ~0.23. Therefore in this paper we show for the first time that the heavy ion optical potentials. The C-C potentials found by this heavy cluster knockout work indicated a hard core in the C-C interaction. At these energies such a hard core is obtained in the RGM formalism while an all through attractive nature (A) obtained from the folding model, as advocated by Wieland et al [15] is proved erroneous.

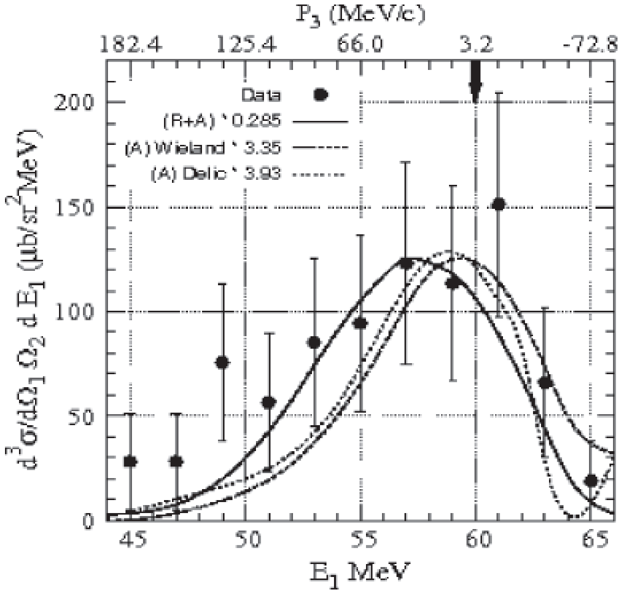

Fig.9 Energy Sharing Spectra analyzed with FR-DWIA.

In summary we find that below ~160 MeV lab. energy the $\alpha - \alpha$ potential has a repulsive core at short distances, whereas beyond this energy the potential becomes fully attractive. Based on the present theory the first ever Heavy Cluster knockout experiment was performed by us. In this it is observed that in the Heavy Ion Cluster Knockout reaction $^{16}O(C,2C)^4He$ at 120 MeV lab. energy there is an additional enhancement in the cross section, which by no means can be resolved by the conventional Zero Range-DWIA formalism. The data has been explained by the Finite Range calculations with the repulsive core in the C-C vertex interaction.

In the present work therefore, the long standing mystery lingering over a period of three decades over the cluster knockout reaction cross sections has been resolved by understanding the physical process. Verification is obtained by executing a novel type of the Heavy Cluster Knockout Experiment at the BARC-TIFR Pelletron-LINAC Facility (PLF).

In future we intend to perform similar heavy cluster knockout reactions on intermediate mass nuclei as well as exotic nuclei (RIB). The knowledge of cluster knockout reaction can also be utilized to observe the behavior of the nucleon-nucleon interaction at short distances or otherwise, if there is a possibility of dibaryon formation at some energy then one should be able to decipher it from the FR-DWIA analyses of the *(p, 2p)* reactions. Similarly one can visualize observing the Δ -resonance in $(\pi, \pi p)$ [16] reaction and the pentaquark, Θ^+ in $(k^+, k^+ n)$ reaction arising as a result of enhanced distortion effects. In heavy ion knockout reactions also one can investigate the short range behavior of the heavy ions involved at the knockout vertex which is rather difficult to observe in the elastic scattering. The present results and conclusions may be very instructive in studies involving *(e, 2e)* reactions on atoms, knockout of atoms from molecules, *(n, 2n)* reactions for neutron multiplication and in many other disciplines involving direct knockout reactions. The present theory also has application in determining the *(n, 2n)* reaction cross section which will be useful in the Accelerator Driven System (ADS).

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